Week 6
MATH 34B
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14.3 Find the solution of the differential equation $y' = (2t+1)^2$ satisfying the initial condition $y(0) = 6$.

$$\frac{dy}{dt} = (2t+1)^2$$

$$y = \int (2t+1)^2 dt = \frac{\frac{1}{3}(2t+1)^3}{2} + C \quad \leftarrow \text{b/c. of } 2t+1.$$

$$y(0) = 6 \Rightarrow \quad 6 = \frac{1}{6}(2(0)+1)^3 + C$$

$$\text{So, } C = 6 - \frac{1}{6}$$

14.6 Find the general solution of the equation $y'' = e^{2t}$.

$$y'' = e^{2t}$$

$$\Rightarrow y' = \int e^{2t} dt = \frac{1}{2} e^{2t} + C_1$$

$$y = \int \frac{1}{2} e^{2t} + C_1 dt$$

$$= \frac{1}{4} e^{2t} + C_1 t + C_2$$

14.9 The number of bees in a forest is growing at a rate of $200 + 10t$ bees per day, t days after being introduced into the forest. If initially 20000 bees are introduced, how many bees are there after 100 days?

b = bees (in numbers)

$$\frac{db}{dt} = 200 + 10t$$

$$b = \int 200 + 10t \, dt$$

$$= 200t + 5t^2 + C$$

$$b(0) = 20{,}000.$$

$$b = 200t + 5t^2 + 20{,}000$$

$$b(100) = 200(100) + 5(100)^2 + 20{,}000$$

16.4 The function y satisfies a differential equation of the form y' = ky for some number k. If you are told that when t = 3 that y is 5 and the rate of change of y is 4 then what is k?

$$y' = ky \Rightarrow y = Ae^{kt}$$

$$\Rightarrow y' = Ake^{kt}$$

At $t = 3$, $y = 5$, $y' = 4$

$$\text{So, } 5 = Ae^{k \cdot 3}$$

$$4 = Ake^{k \cdot 3}$$

$$\Rightarrow k = \frac{4}{5}$$